



VU Research Portal

Multidimensional Locational Decision and Interactive Programming

Nijkamp, P.; Spronk, J.

published in

Organizations: Multiple Agents with Multiple Criteria
1981

document version

Publisher's PDF, also known as Version of record

[Link to publication in VU Research Portal](#)

citation for published version (APA)

Nijkamp, P., & Spronk, J. (1981). Multidimensional Locational Decision and Interactive Programming. In *Organizations: Multiple Agents with Multiple Criteria* (pp. 248-264). Springer.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal ?

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

E-mail address:

vuresearchportal.ub@vu.nl

MULTIDIMENSIONAL LOCATIONAL DECISIONS AND INTERACTIVE PROGRAMMING

Peter Nijkamp

Department of Economics

Free University

Amsterdam, The Netherlands

Jaap Spronk

Department of Economics

Erasmus University

Rotterdam, The Netherlands

ABSTRACT

The paper is devoted to an extension of traditional location theory in two directions. First, the usual assumption of a single cost function will be abandoned by introducing multiple objectives. This gives rise to a multidimensional programming framework for the traditional location models. The paper provides a solution algorithm for the latter problem.

Next, the assumption of a uniform space will be tackled by taking account of discrete candidate-locations. This problem will be solved by means of an adjusted multicriteria analysis.

The solution algorithms for both extensions are based on an interactive strategy, so that the decision-maker may identify the most favourable location in a finite number of steps. The various steps are illustrated by numerical examples.

1. INTRODUCTION

The history of regional science and economic geography has been marked by a great emphasis on location analysis. Much attention has been paid to the problem of where to locate one or more facilities, given a certain objective function for the system at hand. Especially the Weberian approach (characterized by cost minimization, fixed technical coefficients and a homogeneous space) has dominated the history of the location analysis.

Several severe limitations of the Weber model have been tackled during the last decade. For example, the multi-facilities problem has been solved by Cooper [1967] and Scott [1971] among others; also many network location models have addressed these problems. Another shortcoming which has been successfully tackled during the last decade was the problem of fixed technical coefficients (see for an application to neoclassical production functions with input substitution Paelinck and Nijkamp [1976]).

The limitations associated with the assumption of a single uniform cost function and with the assumption of a homogeneous area, however, have received less attention. The present paper will be devoted to overcoming these limitations.

The stringent assumption of a single cost objective will be relaxed by introducing multiple objective functions. This gives rise to a multiobjective optimization (or multidimensional programming) framework (see section 2). The use of (interactive)

multidimensional programming models for the Weber problem will be exposed in section 3 and further illustrated in section 4.

The unrealistic assumption of a homogeneous space will be tackled in section 5, in which by means of a discrete interactive multicriteria analysis the choice problem of distinct candidate-locations will be dealt with. This analysis will be illustrated by means of an example in section 5. A brief evaluation in section 6 will conclude this paper.

2. INTERACTIVE MULTIDIMENSIONAL PROGRAMMING MODELS

It is an evident fact that the locational decisions for facilities (in both the private and the public sphere) are seldom based exclusively on monetary cost objectives, because (1) many aspects of investment decisions cannot be translated into a common monetary denominator (risk, e.g.) and (2) locational decisions usually give rise to external spillover effects or intangibles which can hardly be incorporated in the traditional cost mechanism (environmental pollution, e.g.). Consequently, most locational decisions have to be based on a multiplicity of objective functions, for example, cost minimization, risk minimization, maximization of accessibility, minimization of environmental decay, etc. In formal terms, such a multidimensional programming problem can be formalized as:

$$\min \varphi(\underline{z}), \quad (1)$$

where φ is a 1×1 vector of different objective functions φ_i and \underline{x} a vector of decision arguments. Clearly, the values of \underline{z} are constrained by a feasible area K , i.e.,

$$\underline{z} \in K \quad (2)$$

Normally, the various objectives are mutually conflicting, so that a minimization of the one objective precludes the attainment of a minimum value for the other objectives. In the practice of decision-making, normally a compromise between various extreme options is determined in order to reconcile to a certain extent the conflicting objectives. For locational decisions, for example, the minimum cost location may differ from the minimum risk location (nuclear power plants, e.g.), so that a certain compromise location has to be identified.

This problem of conflicting objectives and compromises has received a lot of attention in recent books on multiobjective optimization ¹⁾.

1) See among others Bell et al. [1977], Blair [1979], Cochrane and Zeleny [1973], Cohon [1978], Van Delft and Nijkamp [1977], Fandel [1972], Guigou [1974], Haimes et al. [1975], Hill [1973], Johnsen [1968], Keeney and Raiffa [1976], Nijkamp [1977, 1979], Starr and Zeleny [1977], Thiriez and Zionts [1976], Wallenius [1975], Wilhelm [1975], and Zeleny [1974, 1976].

A fundamental problem in multiobjective decision theory is the specification of the trade-offs between the conflicting objectives. Clearly, without any information concerning the relative importance of the successive decision criteria, a multiobjective programming model will never lead to an unambiguous solution. The identification of such a compromise solution between conflicting options is normally based on the notion of efficient solutions (Pareto solutions). A vector $\underline{z} \in K$ is an efficient solution if no other feasible solution $\underline{z}^0 \in K$ does exist such that (see Geoffrion [1968], Koopmans [1951], and Kuhn and Tucker [1968]):

$$\left. \begin{array}{l} \varphi_i(\underline{z}^0) \leq \varphi_i(\underline{z}) \quad i=1, \dots, I \\ \text{and} \\ \varphi_i(\underline{z}^0) \neq \varphi_i(\underline{z}) \quad \text{for at least one } i \end{array} \right\} \quad (3)$$

It is easily seen that the ultimate compromise solution should be an element of the class of efficient solutions; any other feasible solution is always dominated by an efficient solution. This is a first (necessary) demarcation criterion for identifying compromise solutions in multiobjective programming models, but it is obviously not sufficient: the set of efficient solutions is very large.

The determination of efficient solutions (the efficiency frontier) may be based (1) on a parametrisation of weights associated with the successive objective functions (see Kuhn and Tucker [1951]) or (2) on a variation of the objective functions included as constraints in a unidimensional programming model (see Haimes et al. [1975]).

As indicated above, the specification of trade-offs is fraught with difficulties. Some approaches to gauge trade-offs (relative priorities) for conflicting objectives are an a posteriori revealed preference approach or an a priori inquiry into the decision-maker's preference structure (based on questionnaires, e.g.). In many practical decision problems, however, these approaches fail to provide relevant or reliable information.

Therefore, recently much attention has been paid to so-called interactive multiobjective optimization models.¹⁾ The common feature of the majority of these interactive models is that they aim at identifying a compromise solution (or sometimes a subset of most preferred solutions) from the set of efficient solutions by means of a stepwise communication process between the analyst and the decision-maker. The analyst provides the decision-maker with information about the set of feasible and efficient solutions and about trade-offs between conflicting solutions, while he also shows him in a stepwise manner one or more compromise solutions.

Then the decision-maker has to express preferences concerning

1) Seen among others Benayoun et al. [1971], Dyer [1972], Fichet [1976], Geoffrion et al. [1972], Monarchi et al. [1976], Nijkamp and Rietveld [1979], Nijkamp and Spronk [1979], Pollack [1976], Price [1976], Rosinger [1979], Steuer and Schuler [1978], Walker [1978] and Wallenius [1975].

these provisional compromise solutions, for example, by rejecting non-satisfactory solutions. Next, the analyst may incorporate this information in his optimization process and suggest a new compromise solution to the decision-maker. This procedure is repeated, until a final satisfactory compromise solution is achieved.

In the present paper, the notion of interactive programming models will be exposed by employing the so-called ideal point approach. The following steps can be distinguished:

- determine the ideal point, i.e. the vector of minimum values of the minimands (provided each objective has to be minimized (see (1) and (2))):

$$\min_{\underline{z} \in K} \left. \begin{array}{l} \varphi_i(\underline{z}), \forall i \\ \end{array} \right\} \quad (4)$$

For normal convex programming problems this minimum is a unique point. The minimum values of (4) are denoted as φ_i^0 and are included in a vector $\underline{\varphi}^0$, called the ideal point. Clearly, $\underline{\varphi}^0$ is not a feasible point, but it may serve as a frame of reference for identifying compromise solutions in the next step.

- determine a provisional compromise point by minimizing the distance between the ideal point and the efficiency frontier:

$$\min_{\underline{z} \in K} \left[\sum_{i=1}^I \left\{ \varphi_i^0 - \varphi_i(\underline{z}) \right\}^2 \right]^{\frac{1}{2}} \quad (5)$$

When the objective functions $\varphi_i(\underline{z})$ have different dimensions, they have to be standardized before (5) can be calculated (see Nijkamp and Rietveld [1979]). The provisional compromise points will be denoted as $\hat{\varphi}_i$.

- ask the decision-maker whether the compromise solutions are satisfactory or not. Let S represent the set of objective functions which are judged to be unsatisfactory, so that the values of these objective functions are to be decreased in value. Consequently, the decision-maker's judgment gives rise to the following additional side-condition:

$$\varphi_i(\underline{z}) \leq \hat{\varphi}_i, \quad \forall i \in S \quad (6)$$

- include the decision-maker's judgment as a constraint in the next stages and repeat the foregoing programming procedure by going back to (4):

$$\min_{\underline{z} \in K} \left. \begin{array}{l} \varphi_i(\underline{z}), \forall i \\ \varphi_i(\underline{z}) \leq \hat{\varphi}_i \end{array} \right\} \quad (7)$$

The above-mentioned interactive procedure is repeated until a final satisfactory and converging solution is obtained (see also Nijkamp [1979]). The advantages of such a procedure are: it provides information to the decision-maker in a stepwise way, it stimulates an active role of the decision-maker, and it avoids the prior specification of trade-offs. The above-mentioned interactive approach will now be applied to a multiobjective Weber problem.

3. AN INTERACTIVE MULTIDIMENSIONAL PROGRAMMING MODEL FOR CONTINUOUS LOCATION PROBLEMS

The general aim of the traditional Weber model is to find the minimum cost location for a firm or a facility, given the location of the input sources and of the markets. Normally, uniform transportation costs over a homogeneous area and fixed technological coefficients are assumed.

If the (known) production volume of the firm concerned is denoted by q and if the successive points of inputs (raw materials, energy etc.) and outputs (final products, intermediate deliveries etc.) are numbered as $1, \dots, n, \dots, N$, the transportation costs T_n from the as yet unknown optimal location of the firm to an arbitrary point n are equal to:

$$T_n(x, y) = c_n d_n(x, y) \quad (8)$$

where: c_n = cost per unit of distance between point n and the facility, given the volume of production.

$d_n(x, y)$ = distance between point n located at (x_n, y_n) and a facility located at (x, y) .

The Euclidean distance d_n is given by:

$$d_n(x, y) = \{(x_n - x)^2 + (y_n - y)^2\}^{\frac{1}{2}} \quad \forall n \quad (9)$$

Then the total transportation costs $T(x, y)$ to all N places are:

$$T(x, y) = \sum_{n=1}^N T_n(x, y) \quad (10)$$

The first-order conditions for a minimum transport cost location are:

$$\frac{\partial T(x, y)}{\partial x} = -\frac{1}{2} \sum_{n=1}^N t_n a_n q (x_n - x) \left\{ (x_n - x)^2 + (y_n - y)^2 \right\}^{-\frac{1}{2}} = 0 \quad (11)$$

Clearly, the first-order conditions with respect to y have an analogous structure. Since it can be proved that the Hessian matrix of second-order derivatives is positive semi-definite (see Paelinck and Nijkamp [1976]), function (10) is convex, and any local extremum of (10) is a unique global minimum. The standard solution algorithm itself consists of an iterative numerical approximation of the optimum (see, for

example, Kuhn and Kuenne [1962] and Cooper [1967]); this algorithm will not be discussed here any further. Extensions of the original Weber problem can also easily be dealt with in the above-mentioned framework (for example, the multi-facility problem, and non-linear transportation costs).

Next, the assumption may be made that the firm at hand has not a single minimum cost objective, but also a wide variety of other relevant objectives¹⁾. This implies that the search for an optimum location is determined by a multiplicity of objective functions. Examples of such additional objective functions are:

- risk minimization for the transport of dangerous inputs (leading to an orientation toward input sources) or dangerous outputs (leading to an orientation towards commodity destinations). In general, one may assume that the risk function is proportional to the distance of shipping the commodities, so that on the basis of linear risk coefficients a risk function analogous to (8) can be derived.
- minimization of energy loss or heat. These losses are also linearly related to the distance to be bridged, so that again a similar expression as (8) can be assumed.
- minimization of pollution (noise, carbon monoxide etc.) from the shipping of commodities between the various places. This leads again to a pollution emission function with linear coefficients per unit of distance and product.

If we assume that the firm has I linear objective functions $\varphi_1, \dots, \varphi_I$ for its locational decisions, the general specification of the i th objective functions is:

$$\min \varphi_i(x, y) = \sum_{n=1}^N T_{ni}(x, y) \quad (12)$$

For each objective function φ_i , the minimum value φ_i^0 can be calculated in the way indicated above, so that also the ideal point φ^0 can be identified. In a similar way, for each objective function φ_i the minimizing co-ordinates (x_i, y_i) can be determined. Because the various objective functions are normally conflicting, the I pairs of co-ordinates $(x_1, y_1), \dots, (x_I, y_I)$ do not coincide.

For example, assuming $N = 5$ and $I = 4$, the following conflict solution might be obtained (see Fig. 1).

It is clear that the ultimate compromise location should fall within the envelope of the points 1, 2, 3, 4 and 5, but its precise location depends on the relative priorities attached to the 4 objective functions.

Normally, the trade-offs associated with the objective functions are not specified in advance by the decision-maker, so that it will be useful to proceed via the interactive procedure set out in section 2. Thus, the following steps have to be carried out:

- calculate the ideal point φ^0 with arguments $\varphi_1^0, \dots, \varphi_I^0$.
- calculate the compromise point $\hat{\varphi}$ with arguments $\hat{\varphi}_1, \dots, \hat{\varphi}_I$ on the basis of the gra-

1) See for other applications to the Weber problem among others El-Shaieb [1978], McGinnis and White [1978], Lawrence and Burbridge [1974], Lawrence and Koch [1975] and Ostrech [1978].

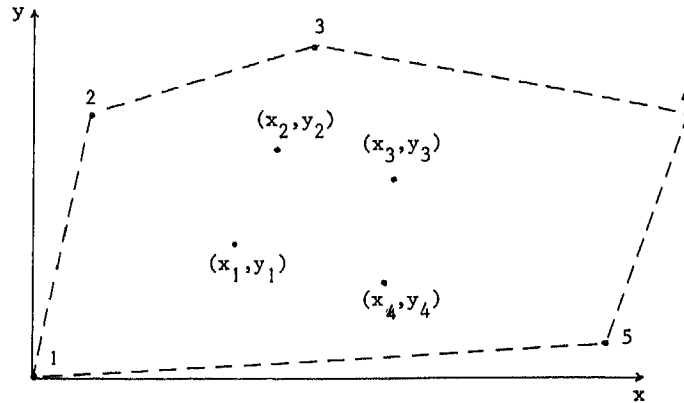


Fig. 1. An illustrative example of a multiobjective location problem with 4 ideal locations.

vity point of the co-ordinates $(x_1, y_1), \dots, (x_I, y_I)$ associated with the ideal point¹⁾.

- ask the decision-maker to express his opinion about the compromise solutions $\hat{\varphi}_1, \dots, \hat{\varphi}_I$, such that he may identify an element $\hat{\varphi}_i$, of the compromise solution which he judges as least satisfactory. This gives rise to the following side-condition:

$$\varphi_i \leq \hat{\varphi}_i, \quad (13)$$

- include (13) as a constraint in the original multiobjective location model, so that one obtains:

$$\left. \begin{array}{l} \min \varphi_i \\ \varphi_i \leq \hat{\varphi}_i \end{array} \right\} \quad \forall i \quad (14)$$

Then the procedure has to be repeated, until finally a satisfactory compromise solution has been found (see for a discussion of convergence properties Fandel [1972]).

There is only one problem left, viz. the solution of (14). This constrained nonlinear programming model cannot be solved directly by means of the algorithm developed for the original unconstrained Weber problem.

Two ways are open to solve the nonlinear multiobjective programming model implied by (14). The first way is to rewrite (14) as a general geometric programming model (see among others Duffin et al. [1967] and Nijkamp [1972]). By defining the elements

1) An alternative procedure may be to minimize the distance metric according to (5).

from the distance relationship d_{ni} included in (12) successively as:

$$d_{ni} = z_{ni}^{\frac{1}{2}} \quad (15)$$

$$z_{ni} = \dot{x}_{ni}^2 + \dot{y}_{ni}^2 \quad (16)$$

and:

$$\left. \begin{aligned} \dot{x}_{ni} &= x_i - x_{ni} \\ \dot{y}_{ni} &= y_i - y_{ni} \end{aligned} \right\} \quad (17)$$

it is easily seen that φ_i has the standard format of the objective function of a geometric programming model. The same holds true for the constraint from (14). Therefore, in principle, one may solve the single-objective version of (14) by means of a standard algorithm for geometric programming models (see for a survey Rijckaert and Martens [1978]). The separate single-objective solutions can then be used to calculate the new compromise points etc.

An alternative way of treating (14) is to use a heuristic algorithm which may be less elegant in itself, but which is closely linked to the single-objective Weber algorithm. Then, the following steps have to be undertaken:

- (1) calculate the unconstrained minima of each objective function φ_i and check whether the constraint specified in (13) is effective. If not, the minimum value of the objective function concerned can be included in the ideal point. Otherwise, proceed to (2).
- (2) if the minimization of a certain φ_i leads to a certain effective constraint, $\varphi_{i'}$, construct the following Lagrange expression with a multiplier λ :

$$L = \varphi_i - \lambda(\varphi_{i'} - \hat{\varphi}_{i'}) \quad (18)$$

- (3) calculate the first-order derivatives with respect to x and y :

$$\left. \begin{aligned} \frac{\delta L}{\delta x} &= \frac{\delta \varphi_i}{\delta x} - \lambda \frac{\delta \varphi_{i'}}{\delta x} = 0 \\ \frac{\delta L}{\delta y} &= \frac{\delta \varphi_i}{\delta y} - \lambda \frac{\delta \varphi_{i'}}{\delta y} = 0 \\ \varphi_{i'} &= \hat{\varphi}_{i'} \end{aligned} \right\} \quad (19)$$

The first two equations of (16) would have a unique solution, when λ would be known, because in that case their structure would formally be equal to (11).

Therefore, the following trial-and-error procedure is reasonable:

- (4) substitute an arbitrary initial solution of λ into (19) and solve the corresponding values of x and y on the basis of the two first equations. Substitute these values into the last constraint of (19) and check whether these values are cor-

rect. If not, adjust λ by means of a grid research procedure until the last constraint is satisfied (see for such a grid procedure also Paelinck and Nijkamp [1976]).

This procedure can be carried out for each active constraint, so that after a series of stages (leading to a reduction of the feasible area) a satisfactory compromise point can be found. When the step sizes of the grid search procedure are not extremely large, a unique solution can normally be found in a limited number of steps. In this way a convergence is usually guaranteed.

4. A NUMERICAL ILLUSTRATION (continuous case)

In this section the use of the interactive model described in section 3 will be illustrated for a continuous location problem. We assume that there are $N=4$ given places which are relevant for a certain firm either as an input source or as a market to be served. The coordinates of these locations are (0,0), (3,0), (0,3) and (2,2) respectively. The problem is to find an optimal location for this firm on the basis of the following four objective functions, which are assumed to be linear with regard to the distances of shipping the commodities:

$$\varphi_1 = 1.d_1 + 3.d_2 + 2.d_3 + 4.d_4 \text{ (transportation costs)}$$

$$\varphi_2 = 2.d_1 + 3.d_2 + 3.d_3 + 2.d_4 \text{ (risk index)}$$

$$\varphi_3 = 4.d_1 + 3.d_2 + 2.d_3 + 1.d_4 \text{ (energy loss)}$$

$$\varphi_4 = 1.d_1 + 2.d_2 + 3.d_3 + 4.d_4 \text{ (pollution index)}$$

The ideal points with respect to each of these objective functions can be found in a straightforward manner by means of the solution algorithm mentioned in section 3. These ideal points have the following coordinates:

objective function	x-coordinate	y-coordinate
1	2.000	1.999
2	1.500	1.500
3	0.527	0.367
4	1.999	2.000

Table 1: Coordinates of 4 ideal points

The first compromise solution is taken as the gravity point of the coordinates of the ideal points. Thus the first compromise can be described by the coordinates (1.506, 1.467); then the corresponding values of the successive objective functions are $\varphi_1 = 15.59$, $\varphi_2 = 18.39$, $\varphi_3 = 19.72$, $\varphi_4 = 15.65$, respectively. Next, it is the

decision-maker's turn. Let us assume that in his view the value of objective function 1 should not exceed a value of $\varphi_1 = 15.59$. It is easily seen that the ideal points for the first and the fourth objective function do not violate this restriction. However, the ideal points of objective functions 2 and 3 do; hence, these two ideal points have to be recalculated subject to the constraint on the value of objective function 1. This can be done through the solution of the Lagrangian in (15) and (16). A small problem may arise because there may be more than one value of 2 (and thus more than one ideal point) which satisfies (16). In that case, the new ideal point was chosen as the one which was closest to the first unconstrained, ideal point. The illustrative results for a series of 7 iterations are given in the following table.

5. AN INTERACTIVE MULTIDIMENSIONAL PROGRAMMING MODEL FOR DISCRETE LOCATION PROBLEMS

In the previous sections a homogeneous area was assumed. In reality, this assumption is often violated, because decision-makers normally make a choice out of a distinct number of candidate-locations. This problem will be discussed in the present section.

Assume again I objective functions to be minimized. Assume also N sources of inputs and destinations of outputs. Next, the assumption is made that the decision-maker has to choose a location from J distinct candidates spatially distributed within a certain relevant location area. Given the known locations of the N input and output places, the J candidate locations, and the distance costs (in a general series) between these locations, one may calculate the values of all I objective functions φ_i ($i=1, \dots, I$) in all J candidate-locations. All these values can be incorporated in a $J \times I$ location impact matrix

	φ_1	φ_I
1								
.								
.								
.								
.								
.								
.								
J								

$[\varphi_{ji}]$

This discrete problem cannot be solved by means of the traditional Weber algorithms. Recently, however, these kinds of distinct evaluation problems have been studied quite extensively in discrete multicriteria analysis (see Van Delft and Nijkamp [1977], e.g.). The specification of trade-offs or weights is here also a great problem, but these discrete decision problems may also be tackled by means of

iteration	coordinates of ideal points				coordinates compromise	value of objective function			
	1	2	3	4		1	2	3	4
1.	(2.000, 1.999)	(1.500, 1.500)	(0.527, 0.367)	(1.999, 2.000)	(1.506, 1.467)	15.59 [*]	18.39	19.72	15.65
2.	do.	do.	(1.581, 1.397)	do.	(1.770, 1.724)	14.63	18.56 [*]	20.96	14.69
3.	(1.770, 1.724)	do.	do.	(1.750, 1.750)	(1.650, 1.593)	15.04	18.43	20.31 [*]	15.12
4.	(1.754, 1.546)	do.	do.	(1.650, 1.593)	(1.621, 1.509)	15.23	18.41	20.02	15.39 [*]
5.	do.	(1.575, 1.546)	(1.553, 1.504)	do.	(1.633, 1.547)	15.15 [*]	18.42	20.15	15.27
6.	do.	(1.627, 1.523)	(1.659, 1.434)	do.	(1.673, 1.524)	15.10	18.44	20.15	15.31 [*]
7. (final)	do.	(1.575, 1.546)	(1.580, 1.516)	do.	(1.640, 1.550)	15.13	18.42	20.17	15.25

Table 2: Results of an illustrative multiobjective location problem.

^{*}denotes that his value is assumed to be an upper limit on the objective function concerned (identified by the decision-maker)

interactive procedures.

The following interactive procedure which runs parallel to the above-mentioned continuous interactive procedure and which has shown its practical relevance in reality (see Van Delft and Nijkamp [1977], and Nijkamp and Spronk [1979]) will be used here:

- determine the ideal point $\underline{\varphi}^{\min}$ with elements φ_i^{\min} - defined as (see also (4)):

$$\varphi_i^{\min} = \min_j \varphi_{ji} \quad \forall i \quad (20)$$

- determine a provisional compromise point $\hat{\underline{\varphi}}$ with elements $\hat{\varphi}_i$ such that this point has a minimum distance with respect to the ideal point¹⁾ (see also (5)).
- ask the decision-maker to judge the provisional compromise solution $\hat{\underline{\varphi}}$ and let him indicate which values of the I objective functions are unsatisfactory. Denote these objective functions by a set S, so that the following constraint arises (see (6)):

$$\varphi_{ji} \leq \hat{\varphi}_i, \quad \forall i \in S \quad (21)$$

- eliminate next all candidate-locations which do not satisfy constraint (21) and repeat the procedure, until finally a satisfactory compromise solution is identified by the decision-maker.

This interactive multicriteria procedure is rather manageable for many practical location problems. It can be adjusted in several ways, for example, by including multiple decision-makers, multiple facilities, and soft (ordinal or qualitative) information (see Nijkamp and Spronk [1979]).

6. A NUMERICAL ILLUSTRATION (discrete case)

In this section a discrete multiobjective location problem will be discussed, which is exactly the same as the example described in section 4, except that there exist now only 20 distinct candidate-locations for the firm. The coordinates of these 20 possibilities are given in the following table:

1) Clearly, when the φ_i 's have different dimensions, one has to standardize them, before the distance metric can be calculated.

location	x-coordinate	y-coordinate	location	x-coordinate	y-coordinate
1	1.0	1.0	11	1.5	1.0
2	2.0	1.0	12	0.5	1.0
3	3.0	2.0	13	1.0	0.5
4	4.0	2.5	14	2.0	1.5
5	2.5	1.5	15	1.5	1.5
6	0.5	0.5	16	0.7	0.4
7	0.4	0.3	17	0.8	0.3
8	0.5	0.6	18	0.9	0.1
9	0.6	0.6	19	0.8	0.8
10	0.7	0.5	20	0.6	0.5

Table 3: Coordinates of 20 candidate-locations

From these data, the following 20 x 4 location impact matrix P can be calculated (see 12):

location	φ_1	φ_2	φ_3	φ_4
1	24.03	17.23	14.04	24.03
2	24.90	21.93	20.79	25.49
3	27.36	29.65	31.47	27.69
4	32.21	37.62	41.04	32.58
5	26.04	25.67	26.04	26.47
6	28.41	19.15	12.41	28.41
7	28.12	17.43	11.62	28.26
8	28.31	17.63	12.45	28.16
9	28.22	18.04	12.69	28.29
10	27.77	17.37	11.97	28.05
11	24.27	19.51	17.59	24.66
12	26.16	16.93	13.74	24.55
13	23.72	16.52	12.93	24.34
14	26.33	24.62	23.76	27.02
15	25.23	21.34	20.20	25.26
16	24.25	15.93	11.59	24.67
17	24.03	15.97	11.62	24.74
18	26.36	17.30	12.22	27.50
19	24.22	16.69	13.39	24.22
20	24.50	15.95	11.67	24.65

Table 4. An illustrative location impact matrix for 20 points.

According to (17), the ideal point can be identified as

$$\underline{\varphi}^{\min} = (23.72, 15.93, 11.59, 24.03) \quad (22)$$

After a standardization of the location impact matrix by means of

$$w_{ji} = \frac{\max_{k=1, \dots, J} (\varphi_{ki}) - \varphi_{ji}}{\max_{k=1, \dots, J} (\varphi_{ki}) - \min_{k=1, \dots, J} (\varphi_{ki})}, \quad \forall j, \forall i \quad (23)$$

we can calculate the first compromise location, i.e. the location which, according to (5), has the smallest distance with respect to the ideal point. This first compromise thus becomes location 13. Let us assume that the decision-maker regards the value of the third objective function in this compromise as an upper limit which may not be surpassed. This leads to the following constraint:

$$\varphi_{j3} \leq 12.93 \quad (24)$$

This means that we can eliminate all locations which do not meet this requirement, i.e. locations 1, 2, 3, 4, 5, 11, 12, 14, 15 and 19. By repeating the previous procedure, the new ideal point becomes:

$$\underline{\varphi}^{\min} = (23.72, 15.93, 11.59, 24.34) \quad (25)$$

Then the second compromise solution is location 16. The decision-maker is then assumed to restrict the value of the first objective function as follows:

$$\varphi_{ji} \leq 24.25 \quad (26)$$

By doing so, the locations 6, 7, 8, 9, 10, 18 and 20 can be dropped, so that only locations 13, 16 and 17 are left. The new ideal point is now:

$$\underline{\varphi}^{\min} = (23.72, 15.93, 11.59, 24.34) \quad (27)$$

The compromise solution which then results is location 17. Let us assume that the decision-maker prefers a value of the second objective function below 15.97. Then location 13 can also be eliminated. The only locations left are 16 and 17. The new compromise solution is location 16, which we assume to be accepted by the decision-maker.

7. CONCLUSION

The locational decision problems discussed in the previous sections were based on the traditional Weber analysis. Their aim was to tackle some of the severe limitations of the Weber approach. A multiobjective programming framework appeared to provide a significant extension of the traditional restrictive Weber analysis. It also appeared that discrete multicriteria analysis was another useful tool for locational analysis, especially for distinct location problems. The general conclusion is that, in particular, the interactive variants of multiobjective optimization models offer many opportunities for a more satisfactory and modern locational analysis.

REFERENCES

- Bell, D.E., R.L. Keeney and H. Raiffa, Conflicting Objectives in Decisions, John Wiley, New York, 1977.
- Benayoun, R., J. de Montgolfier, J. Tergny and O.Larichev, Linear Programming with Multiple Objective Functions: Step Method (STEM), Mathematical Programming, Vol. 1, 1971, pp. 366-375.
- Blair, P.D., Multiobjective Regional Energy Planning, Martinus Nijhoff, The Hague/Boston, 1979.
- Cochrane, J.L. and M.Zeleny (eds.), Multiple Criteria Decision Making, University of South Carolina Press, Columbia, 1973.
- Cohon, J.L., Multiobjective Programming and Planning, Academic Press, New York, 1978.
- Cooper, L., Solution of Generalized Locational Equilibrium Models, Journal of Regional Science, vol. 7, no. 1, 1967, pp. 1-18.
- Delft, A. van and P.Nijkamp, Multicriteria Analysis and Regional Decision-Making, Martinus Nijhoff, The Hague/Boston, 1977.
- Duffin, R.J., E.L.Peterson, and C.Zener, Geometric Programming, Wiley, New York, 1967.
- Dyer, J.S., Interactive Goal Programming, Management Science, Vol. 19/1, 1972, pp. 62-70.
- El-Shaieb, The Single Source Weber Problem-Survey and Extensions, Journal of the Operational Research Society, Vol. 29/5, 1978, pp.469-476.
- Fandel, G., Optimale Entscheidung bei Mehrfacher Zielsetzung, Springer Verlag, Berlin 1972.
- Fichefet, J. GPSTEM: An Interactive Multi-objective Optimization Method, in A.Prākopa (ed.), Progress in Operations Research (Vol. 1), North Holland, Amsterdam, 1976.
- Geoffrion, A.M., Proper Efficiency and the Theory of Vector Maximization, Journal of Mathematical Analysis and Applications, Vol. 22, 1968, pp.618-630.
- Geoffrion, A.M., J.S.Dyer, and A.Feinberg, An Interactive Approach for Multi-criterion Optimization with an Application to the Operation of an Academic Department, Management Science, Vol. 19, 1972, pp. 357-368.

- Guigou, J.L. Analyse des Données et Choix à Critères Multiples, Dunod, Paris, 1974.
- Haimes, Y.Y., W.A.Hall and H.T.Freedman, Multi-objective Optimization in Water Resource Systems, Elsevier, Amsterdam, 1975.
- Hill, M., Planning for Multiple Objectives, Monograph Series no. 5, Regional Science Research Institute, Philadelphia, 1973.
- Johnsen, E., Studies in Multi-objective Decision Models, Monograph No. 1, Economic Research Center in Lund, Lund, Sweden, 1968.
- Keeney, R.L., and H. Raiffa, Decision Analysis with Multiple Conflicting Objectives, John Wiley, New York, 1976.
- Koopmans, T.C., Activity Analysis of Production and Allocation, John Wiley, New York, 1951.
- Kuhn, H.W., and R.E.Kuenne, An Efficient Algorithm for the Numerical Solution of the Generalized Weber Problem in Spatial Economics, Journal of Regional Science, Vol. 4, no. 2, 1962, pp. 21-33.
- Kuhn, H.W., and A.W.Tucker, Nonlinear Programming, Readings in Mathematical Economics, (P. Newman, ed.), The Johns Hopkins Press, Baltimore, 1968, pp. 3-14.
- Lawrence, K.D., J.J.Burbridge, and J.J. March, A Multiple Goal Weber Problem for the Location of Regional Service Offices in a Property and Casualty Insurance Company, Proceedings of the 1974 Northeast Regional Meeting of the American Institute of Decision Sciences, Philadelphia, 1974.
- Lawrence, K.D., and H.B.Koch, A Multiple Goal Linear Programming Model for Capital Management Funding, Producing or Purchasing and Scheduling, Proceedings of the 1975 Systems Engineering Conference AIIE, Las Vegas, 1975.
- McGinnis, L.F., and J.A.White, A Single Facility Rectilinear Location Problem with Multiple Criteria, Transportation Science, Vol. 12/3, 1978, pp. 217-231.
- Monarchi, D.E., J.E. Weber and L.Duckstein, An Interactive Multiple Objective Decision-Making Aid Using Nonlinear Goal Programming, Multiple Criteria Decision-Making (M. Zeleny, ed.), Springer Verlag, Berlin, 1976, pp. 235-253.
- Nijkamp, P., Planning of Industrial Complexes by means of Geometric Programming, Teakfield, Farnborough, 1972.
- Nijkamp, P., Theory and Application of Environmental Economics, North Holland Publ. Co., Amsterdam, 1977.
- Nijkamp, P., and P.Rietveld, Conflicting Social Priorities and Compromise Social Decisions, Analysis and Decision in Regional Planning (I Cullen, ed.), Pion, London, 1979, pp. 153-177.
- Nijkamp, P., Multidimensional Spatial Data and Decision Analysis, John Wiley, New York/London, 1979.
- Nijkamp, P., and J.Spronk, Analysis of Production and Location Decisions by means of Multicriteria Analysis, Engineering and Process Economics, Vol. 4, 1979, pp. 285-302.
- Ostresh, L.M., On the Convergence of a Class of Iterative Methods for Solving the

- Weber Location Problem, Operations Research, Vol. 26/4, 1978, pp. 597-609.
- Paelinck, J.H.P., and P.Nijkamp, Operational Theory and Method of Regional Economics, Saxon House, Farnborough, 1976.
- Pollack, M., Interactive Models in Operations Research - An Introduction and Some Future Research Directions, Comput. & Ops. Res., Vol. 3/4, 1976, pp. 305-312.
- Price, W.L., An Interactive Objective Function Generator for Goal Programming, Multiple Criteria Decision Making, (H. Thiriez and S. Zionts, eds.), Springer Verlag, Berlin, 1976, pp. 147-157.
- Rosinger, E.E., Interactive Algorithm for Multiobjective Optimization (Revised Version), Technical Report No. 138, Technion-Israel Institute of Technology, Haifa, Israel, Nov. 1978.
- Rijckaert, M.J. and X.M. Martens, Comparison of Generalized Geometric Programming Algorithms, Journal of Optimization Theory and Applications, Vol. 26, no. 2, 1978, pp. 205-272.
- Scott, A.J., Combinational Programming, Spatial Analysis and Planning, Methuen, London, 1971.
- Starr, M.K., and M.Zeleny (eds.), Multiple Criteria Decision Making, North Holland Publ. Co., Amsterdam, 1977.
- Steuer, R.E., and A.T.Schuler, An Interactive Multiple-Objective Linear Programming Approach to a Problem in Forest Management, Operations Research, Vol. 26/2, 1978, pp. 254-269.
- Thiriez, H., and S.Zionts (eds.), Multiple Criteria Decision Making, Springer Verlag, Berlin, 1976.
- Walker, J., An Interactive Method as an Aid in Solving Bicriterion Mathematical Programming Problems, Journal of the Operational Research Society, Vol. 29/9, pp. 915-922.
- Wallenius, J., Interactive Multiple Criteria Decision Methods, Helsinki School of Economics, Helsinki, 1975.
- Wallenius, J., Comparative Evaluation of Some Interactive Approaches to Multicriterion Optimization, Management Science, Vol. 21/12, 1975, pp. 1387-1396.
- Wilhelm, J., Objectives and Multi-Objective Decision Making under Uncertainty, Springer Verlag, Berlin, 1975.
- Zeleny, M. (ed.), Multiple Criteria Decision Making, Springer Verlag, Berlin, 1976.
- Zeleny, M., Linear Multi-objective Programming, Springer Verlag, Berlin, 1974.